A Synthesis of Mathematical and Cognitive Performances of Students With Mathematics Learning Disabilities

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Abstract
The purpose of this study was to synthesize the findings from 23 articles that compared the mathematical and cognitive performances of students with mathematics learning disabilities (LD) to (a) students with LD in mathematics and reading, (b) age- or grade-matched students with no LD, and (c) mathematical-ability-matched younger students with no LD. Overall results revealed that students with mathematics LD exhibited higher word problem-solving abilities and no significant group differences on working memory, long-term memory, and metacognition measures compared to students with LD in mathematics and reading. Findings also revealed students with mathematics LD demonstrated significantly lower performance compared to age- or grade-matched students with no LD on both mathematical and cognitive measures. Comparison between students with mathematics LD and younger students with no LD revealed mixed outcomes on mathematical measures and generally no significant group differences on cognitive measures.

Keywords
cognitive performance, learning disabilities in mathematics and reading, mathematics learning disabilities, mathematical performance

Approximately 5% to 8% of school-age children exhibit mathematics learning disabilities (MLD; Badian, 1983; Geary, 2004; Murphy, Mazzocco, Hanich, & Early, 2007) with persistent early difficulties in number sense (e.g., understanding number magnitude) and basic fact retrieval strategies (Geary, 2011; Jordan, Hanich, & Kaplan, 2003). In addition, according to the reauthorized Individuals with Disabilities Education Improvement Act (IDEIA, 2004), specific learning disabilities (LD) may be manifested in mathematics calculations and/or mathematics problem solving if inadequate mathematics performance persists despite appropriate interventions.

Increasingly, researchers have sought to better understand the nature of MLD through investigations of the mathematical and cognitive performances of students with MLD compared to students who have reading disabilities, are low achievers (LA), and are typically achieving (e.g., Geary, 1993, 2004, 2011; Swanson & Jerman, 2006). Through these studies, the performance of students with MLD has been examined in an attempt to discern the nuances of mathematics difficulties, which conceivably can inform the development of remedial interventions in an attempt to influence students’ prognoses for improved mathematics achievement. Therefore, it is important to review the literature regarding the mathematical and cognitive performances of students with MLD to better inform the field about the learning characteristics of these students. Several reviews have been conducted to examine the performance of students with MLD in relation to other groups of students. We provide an overview of the findings of these studies and their limitations as a framework for the focus of this present synthesis.

Previous Reviews on the Mathematical and Cognitive Performances of Students With MLD
Several studies were located that examined the mathematical and/or cognitive variables conceivably associated with MLD. In one review, Geary (1993) examined the core cognitive and neuropsychological deficits underlying MLD. From a cognitive perspective, his findings revealed that many kindergarten and first-grade students with MLD showed an immature understanding of counting and were delayed in the development of their computational skills (e.g., difficulties in regrouping in addition and counting), spending more time on solving arithmetic problems than

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typically achieving students. In addition, Geary noted that kindergarten and first-grade students with MLD demonstrated persistent deficits in fact retrieval, frequent computational errors, and unsystematic retrieval speeds. From a neuropsychological perspective, Geary found that elementary students with MLD had “damage to the posterior regions of the right hemisphere leading to difficulty in visuospatial representations of numerical information (Spatial acalculia) or to the lesions of the left hemisphere with deficits in reading and writing of numbers (Alexia and agraphia for numbers)” (p. 352). He posited that MLD and reading learning disabilities (RLD) might co-occur because of a common underlying neuropsychological deficit, reflecting a general deficit in the representation or retrieval of information from semantic memory.

In a later review, Geary (2004) examined deficits for different groups particularly related to cognitive performance. His findings showed that in first and second grades, students with LA in both mathematics and reading, particularly students with MLD and RLD, had deficits in working memory and aspects of counting such as the use of counting strategies to solve arithmetic calculations. Geary noted that students with MLD and RLD and students with MLD only relied on finger counting in first and second grades; yet students with RLD only and typically achieving students shifted from using finger counting to verbal counting and automatic retrieval in the second grade. He concluded that the use of finger counting among students with MLD reduced the cognitive load for working memory with the counting process. Moreover, students with MLD showed more errors in retrieving arithmetic facts from long-term memory than their typical peers.

In another review, Swanson and Jerman (2006) quantitatively analyzed findings from studies published between 1983 and 2002 on the cognitive performance of kindergarten through adolescent students with MLD compared to typically achieving students, students with MLD, and students with comorbid disorders in ten areas including “literacy-reading, problem solving-verbal, naming speed, problem solving-visual-spatial, long-term memory, short-term memory-words, short-term memory-numbers, working memory-verbal, working memory-visual-spatial, and attention” (p. 255). Students with MLD demonstrated higher cognitive deficits “on measures of verbal problem solving, naming speed, verbal working memory, visual-spatial working memory, and long-term memory” (p. 265) compared with age-matched typically achieving peers. Students with MLD had less cognitive deficits than students with comorbid disorders “on measures of literacy, visual-spatial problem solving, long-term memory, short-term memory, and verbal working memory” (p. 265). However, there was no distinctive difference between students with MLD and students with RLD on most measures; only on naming speed and visual-spatial working memory measures, thus, students with MLD had higher cognitive deficit than students with RLD. Swanson and Jerman concluded that MLD was persistent across younger (younger than 8.5 years) and older ages (8.5 years or older), and the severity of mathematics and intellectual levels was minimally related to cognitive performance.

Finally, Geary (2010, 2011) comprehensively reviewed the performance of students with MLD, in mainly kindergarten through early elementary school ages, compared to those with LA based on cognitive, neuropsychological, and genetic aspects, emphasizing the progress in better understanding MLD since 1993. Geary (2010) found ample evidence of developmental delays in number sense (e.g., “exact quantity of objects and approximate magnitude of larger quantities”; p. 130) among students with MLD but less so for LA students. However, even though many LA students did not have deficits in working memory in general, some of them exhibited deficits in “inhibitory control subcomponents of the central executive working memory” (p. 259). Subsequent findings corroborated previous results confirming that many kindergarten through early elementary school students with MLD displayed deficits in number sense of numerical magnitude comparable to students with LA. However, students with MLD had more persistent deficits in working memory areas (i.e., phonological, visual-spatial, and central executive), whereas students with LA chiefly had deficits in the inhibition of central executive processing resulting in slow processing speed.

Together these five studies (Geary, 1993, 2004, 2010, 2011; Swanson & Jerman, 2006) examined the cognitive performances of students with MLD compared to students with MLD and RLD, students with MLD only, LA, and typically performing students of similar and older ages. Overall, students with MLD exhibit developmental delays in computational skills, compared to low and typically achieving students, which are manifested in deficits in fact retrieval, calculation errors, and retrieval speed. MLD is persistent across younger and older ages with deficits in working and long-term memory. Students with MLD and students with MLD and RLD may share an underlying neuropsychological deficit in the representation of information from semantic memory and working memory. However, students with MLD and RLD develop more appropriate counting strategies for solving facts than do students with MLD only who tend to rely on the use of finger counting to solve arithmetic calculations.

Collectively, these reviews offer much insight about MLD; however, the findings are limited to mainly early age groups of kindergarten through elementary school students regarding early numeracy system and cognitive characteristics. Therefore, there is a need for synthesizing the learning characteristics of students with MLD across broader elementary and secondary school years to better understand how MLD is manifested in early as well as later years.
This present synthesis targeted both the mathematical and cognitive performances of students with MLD and extended the previous meta-analysis by Swanson and Jerman (2006) with more rigorous inclusion criteria. Studies with students not specifically identified as having MLD (e.g., Garnett & Fleischner, 1983; Siegel & Ryan, 1988) were excluded. Furthermore, this synthesis set criteria for participants’ grades from 2nd grade to 12th grade. Last, this study included the comparison group of younger students with no LD matched on the mathematical ability of students with MLD. This group comparison was extended by the previous meta-analysis by Swanson and Jerman (2006) with more rigorous inclusion criteria. Participants were identified as having MLD, MLD/RLD, and NLD-age or NLD-grade. All studies included an operational definition of MLD (Gersten et al., 2009) including studies with school identified students with MLD who had a mathematics goal on their individualized education program (IEP). Studies that focused on students with only mathematics difficulties (e.g., Jordan et al., 2003) were excluded because this present study focused on comparing the performances of different groups of students instead of the effects of interventions or the identification of MLD. Articles focused on mathematical and cognitive performances of students with MLD. Articles that targeted neuroscience, pediatrics, and emotional and behavioral aptitudes were excluded.

The purpose of this study was to synthesize the literature comparing the mathematical and cognitive performances of (a) students with MLD and students with MLD and RLD (MLD/RLD), (b) students with MLD and students with no LD matched on the same age or grade (NLD-age or NLD-grade), and (c) students with MLD and younger students with no LD matched on mathematical ability (NLD-ability). The specific research questions for this study were as follows:

1. How do students with MLD differ from those with LD in mathematics and reading as measured by mathematical and cognitive measures (MLD vs. MLD/RLD)?
2. How do students with MLD differ from those with no LD matched on the same age or grade of students with MLD as measured by mathematical and cognitive measures (MLD vs. NLD-age or NLD-grade)?
3. How do students with MLD differ from younger students with no LD matched on the mathematical ability of students with MLD as measured by mathematical and cognitive measures (MLD vs. younger NLD-ability)?

**Method**

**Literature Search Procedures**

All studies published between 1975 and 2011 that focused on the mathematical and cognitive performances of students with MLD were included. Thus, we extended the search years beyond the meta-analysis conducted by Swanson and Jerman (2006) and initiated the search from 1975 to capture all of the articles that met our criteria.

This search was conducted using the following three steps. First, an online search was performed, using electronic databases (i.e., ERIC, JSTOR, and PsycINFO). A list of keywords (cognitive performance, learning disabilities in mathematics and reading, mathematics learning disabilities, and mathematical performance) and their combinations were used to identify potential studies in the literature. Next, a search for relevant abstracts was conducted in the following journals: Cognitive Development, Exceptional Children, Journal of Educational Psychology, Journal of Experimental Child Psychology, Journal of Learning Disabilities, Learning Disabilities: A Multidisciplinary Journal, Learning Disabilities Research & Practice, Learning Disability Quarterly, Remedial and Special Education, and Journal of Special Education. Last, a manual search of the references of each identified article was conducted for relevant articles.

This search resulted in 538 studies. After reviewing the title, keywords, and abstracts of these 538 studies, 105 studies were selected for further review. Of these 105 studies, the studies that met the following inclusion criteria were selected for in-depth analysis:

- Participants were identified as having MLD, MLD/RLD, and NLD-age or NLD-grade. All studies included an operational definition of MLD (Gersten et al., 2009) including studies with school identified students with MLD who had a mathematics goal on their individualized education program (IEP). Studies that focused on students with only mathematics difficulties (e.g., Jordan et al., 2003) were excluded because this present study focused on investigating the mathematical and cognitive performances of students with MLD.
- Participants were in 2nd (e.g., approximately 8 years old) through 12th grade because these are the grades when typically students are identified with MLD.
- Articles were published in English and in peer-reviewed educational journals between 1975 and 2011. The year 1975 was selected because this is the year the category of LD was first included in the Education for All Handicapped Children Act (Pub. L. No. 94-142).
- Articles focused on mathematical and cognitive performances of students with MLD. Articles that targeted neuroscience, pediatrics, and emotional and behavioral aptitudes were excluded.
- Articles that focused on mathematics interventions and the predictors of MLD were excluded because this present synthesis focused on comparing the performances of different groups of students instead of the effects of interventions or the identification of MLD through longitudinal studies.
- Articles must have included technically adequate measures (e.g., using commercial measures or reporting the reliability and validity of researcher-developed measures).
• Measures assessed the mathematical and cognitive performances of students with LD and their comparison groups (e.g., students with MLD, students with MLD/RLD, students with NLD-age or NLD-grade, and younger students with NLD-ability).
• Articles had to include number of participants and statistical information (e.g., participant numbers, means, and standard deviations) needed to calculate standardized mean differences (SMDs).

Of the 105 studies, 23 (21.90%) met the criteria for inclusion in this synthesis; 82 studies were excluded for the following reasons: (a) they did not measure mathematical performance or cognitive performance \((n = 27)\); (b) they included students with mathematics difficulties, but not specifically students with LD in mathematics \((n = 26)\); (c) there were no disaggregated data for target students with LD in mathematics \((n = 12)\) and comparison students (e.g., students with LD in reading and mathematics or students with LD in reading; \(n = 6\)); and (d) they focused on the identification or predictors of LD in mathematics \((n = 11)\).

**Data Analysis**

**Coding of the studies.** A total of 23 studies, which qualified for the study, were identified and coded for the number of participants, gender, ethnicity, socioeconomic status, achievement measures and scores (i.e., mathematics and reading), intelligence scores (i.e., IQ), age and grades across four different groups of students with MLD, students with MLD/RLD, students with NLD-age or NLD-grade, and younger students with NLD-ability. Specifically, information on the mathematical and cognitive performances of the participants, measures used in each study, and the results of each study were also identified and coded. Two raters independently coded 26% \((n = 6)\) of the included studies. An interrater reliability of approximately 96% was established using the following formula: number of agreed variables / number of total variables × 100%. The two raters reached consensus on conflicting items.

Based on the coding results of the mathematical performance, four mathematical variables (i.e., mathematical calculations, word problem solving, arithmetic strategies, and number sense) emerged, which are included in the Common Core State Standards for Mathematics (Council of Chief State School Officers & National Governors’ Association, 2010). If a study had a standardized mathematics test that focused on arithmetic computations, it was coded as mathematical calculations. If the same study targeted on certain counting strategies in solving mathematical problems, it was also coded as arithmetic strategies. Also, based on the coding information of cognitive performance, four cognitive variables (i.e., working memory, processing speed, long-term memory, and metacognition) were identified.

**Mathematical variables.** The mathematical calculation variable includes the completion of single-digit addition, subtraction, and multiplication facts (Vukovic & Siegel, 2010). The word problem-solving variable consists of accuracy regarding arithmetic word problems and real-world problem-solving measures (Swanson & Jerman, 2006). The arithmetic strategies variable includes arithmetic computational strategies including counting fingers, verbal counting, and retrieval (Geary & Brown, 1991; Geary, Hoard, Byrd-Craven, & DeSoto, 2004). The number sense variable comprises abilities to comprehend the number of objects in a set without counting, understand the quantities of small collections of objects and symbols (Geary, 2011), and comprehend the order of rational numbers (Mazzocco & Devlin, 2008).

**Cognitive variables.** Working memory is one variable that includes the phonological loop, which is responsible for the storage of verbal information (e.g., word, digit forward span, and word forward span; Passolunghi, 2011; Zheng, Swanson, & Marcouilides, 2011); visual-spatial, which is responsible for the storage of mental images (e.g., visual matrix, picture sequence, and mapping and directions; Swanson, 1993, 1994); and central executive, which coordinates and interacts with the above two working memory components (e.g., listening sentence span, digit/sentence span, backward digit span, and operation span; Mabbott & Bisanz, 2008; Zheng et al., 2011). The processing speed variable refers to processing relatively simple information quickly and easily, including counting speed, speed of retrieval (Temple & Sherwood, 2002), and speed of rapid naming of letters, numbers, and objects (Geary, Hoard, Nugent, & Byrd-Craven, 2008; Swanson & Jerman, 2006). The long-term memory variable is the ability to retrieve general factual knowledge of numbers, words, or objects from their long-term memory (Mussolin & Noël, 2008). Long-term memory also includes the ability to retrieve basic facts (Geary, 2004). The metacognition variable consists of skills of prediction and evaluation (Garrett, Mazzocco, & Baker, 2006).

**Computation of standardized mean differences.** Data from the 23 studies were analyzed in the following ways. SMDs for each study were calculated using Hedges’s \(g\) to examine the magnitude of the mathematical and cognitive performances across three comparison groups (students with MLD vs. students with MLD/RLD, students with MLD vs. students with NLD-age or NLD-grade, and students with MLD vs. younger students with NLD-ability). SMDs were defined as the difference between the mean performances of the two groups divided by the pooled within-group standard deviation (Cooper, Hedges, & Valentine, 2009; What Works Clearinghouse, 2008). According to Cohen’s (1988) criteria, effect sizes (SMDs here) around .80 are considered...
large, those around .50 are medium, and those less than .20 are small. SMDs would explain to what degree students with MLD are distinctive from other comparison groups as measured by mathematical and cognitive measures. We did not combine SMDs across measures and across studies and did not consider any clustering factors because the purpose of this article was to synthesize SMDs within each study; we reported SMDs according to the measures that each study used.

**Descriptive analysis.** This synthesis also described the mathematical and cognitive performances of participants, while supplementing the quantitatively analyzed data using SMDs. First, according to the four mathematical variables of mathematical calculations, word problem solving, arithmetic strategies, and number sense, students with MLD’s mathematical performance were compared to students with MLD/RLD, students with NLD-age or NLD-grade, and younger students with NLD-ability. Next, students with MLD’s cognitive performance in working memory, processing speed, long-term memory, and metacognition aspects were descriptively compared to students with MLD/RLD, students with NLD-age or NLD-grade, and younger students with NLD-ability. To provide additional support for the SMDs that we calculated, we reported the statistical significance of the group difference only if the included studies reported them; studies used different $p$ values of .05 or .01, and not all studies reported their significance.

**Results**

**Features of Studies and Participants**

Of the 23 studies, 9 were conducted in the United States, 13 were conducted outside the United States, and 1 was conducted in both the United States and another country. Publication years ranged from 1985 to 2011, with an average year of 2001; 13 studies were published between 2000 and 2011, 8 were between 1990 and 1999, and 2 were between 1985 and 1989.

The total number of participants was 2,081: 715 students with MLD, 192 students with MLD/RLD, 923 students with NLD-age or NLD-grade, and 251 younger students with NLD-ability. In addition, of the 23 studies, most ($n = 18$) included elementary school students in second through fifth grade and 5 others included middle school students in sixth through eighth grade. No studies included high school students. Of the 23 studies, most ($n = 21$) reported the gender of participants (1,011 boys and 889 girls). Of those 21, 19 studies gave gender information for all participants, including comparison groups, but 2 (Desoete & Grégoire, 2006; Fuchs & Fuchs, 2002) did not give comparison group information. In all, 15 did not report both the ethnicity and socioeconomic status of participants, 6 reported only ethnicity (301 White, 39 Black, 36 Hispanic, 2 Asian, and 30 Other), and 2 reported only socioeconomic status (10 free or reduced lunch and 384 in the comparable level).

**Mathematical Performance**

**Students with MLD versus students with MLD/RLD (RQ1).** Of the 23 studies, seven (Desoete & Roeyers, 2002; Fletcher, 1985; Fuchs & Fuchs, 2002; Geary, Hamson, & Hoard, 2000; Räsänen & Ahonen, 1995; Rousselle & Noël, 2007; Schuchardt, Maehler, & Hasselhorn, 2008) compared the mathematical calculations of students with MLD versus students with MLD/RLD. One study (Fuchs & Fuchs, 2002) compared the two groups of students on word problem solving and another (Räsänen & Ahonen, 1995) compared them on arithmetic strategies. No study compared number sense concepts between students with MLD and students with MLD/RLD.

**Mathematical calculations.** The seven studies that examined differences in mathematical calculations based on standardized test results between groups of students with MLD and students with MLD/RLD showed that there were no significant group differences, in general. The two groups of both elementary and middle school students performed comparably on standardized mathematics tests that included mathematical calculation problems (SMD $M = 0.12,$ $p s > .05$). Standardized mathematical calculation tests in the different studies included the Kortrijksche Rekentest [KRT]; Cracco et al., 1995), which measures mental computation (e.g., “129 + 879 = __”) and number system knowledge (e.g., “add three tens to 61 and you have __”); the Wide Range Achievement Test—Revised (WRAT-R; S. Jastak & Wilkinson, 1984); the RMAT (Räsänen, 1992); the Deutscher Mathematiktest für vierte Klassen (DEMAT 2+; Krajewski, Liehm, & Schneider, 2004); the DEMAT 3+ (Roick, Gößler, & Hasselhorn, 2004); the DEMAT 4 (Gößler, Roick, & Hasselhorn, 2006), which included computation problems, word problems, and geometry problems (Schuchardt et al., 2008); the Wide Range Achievement Test (WRAT; J. Jastak & Jastak, 1978) Arithmetic subtest; the Test of Computational Fluency; the Wechsler Individual Achievement Test (WIAT; Wechsler, 1992) Mathematics Reasoning; and the Arabic number writing, untimed subtraction, and timed additions.

**Word problem solving.** Students with MLD in Grade 4 performed better than their peers with MLD/RLD on all problem-solving measures (Fuchs & Fuchs, 2002). Specifically, on the arithmetic story problem test that presented “essential and brief text with one-step number facts for solutions including change, combination, comparison, and equalization problem types” (p. 568), students with MLD scored higher than students with MLD/RLD on both
operations (SMD = 0.88) and problem solving dimensions (SMD = 0.79). On the complex story problem test that included “shopping list problems, halving problems, pictograph problems, and buying bag problems” (p. 569) and the real-world problem solving test that included “contextually realistic situations with irrelevant numbers” (pp. 568–569), the group mean differences were smaller on operations dimension (SMD \( M = 0.35 \)) compared to those on the problem solving dimension (SMD \( M = 0.96 \)).

In addition, when comparing performances on the three different measures with different levels of challenging problems, both students with MLD and students with MLD/RLD showed better arithmetic story problem solving that required less challenging tasks than on complex story problems or real-world problem solving tasks that required more difficult problems in terms of “words, sentences, words per sentence, and verbs” (p. 568).

**Arithmetic strategies.** Students with MLD in Grades 3 through 6 scored significantly higher than their peers with MLD/RLD on the arithmetic fact error task, making significantly fewer arithmetic fact errors than students with MLD/RLD (SMD = 0.84; Räsänen & Ahonen, 1995). The two groups’ means, however, did not significantly differ on other types of arithmetic errors (SMD \( M = -0.10, \text{ps} > .05 \)). The comparable arithmetic error tasks included computational errors (e.g., “incorrect addition, subtraction, or multiplication”; p. 282), random errors (“no logic found between the problem and the solution”; p. 282), fact errors (“computational errors in multiplication tables”; p. 282), wrong operation (“wrong execution of calculation, but with a correct computation”; p. 282), digit errors (“failure to correctly use a number or digits present in the problem when computing them”; p. 282), rule errors (“confusion of the borrowing or carry process”; p. 282), and algorithm errors (“failure to execute correct steps required even though a correct operation is initiated”; p. 282).

To summarize, the overall findings showed that students with MLD showed better word problem solving and arithmetic fact strategy performance than students with MLD/RLD, but no statistically significant group differences were observed on mathematical calculation measures. Although students with MLD performed better than students with MLD/RLD on word problem solving in general, the gap between the two groups was larger on problem-solving tasks that included more text-related components than arithmetic components. This finding showed that the two groups achieved comparably high scores on story problem equation questions, yet on complex and real-world story problems, demanding more reading abilities, students with MLD performed relatively higher than students with MLD/RLD.

**Students with MLD versus students with no LD matched on the same age or grade of students with MLD (RQ2).** Of the 23 studies, 17 studies (Desoete & Roevers, 2002; Fletcher, 1985; Garrett et al., 2006; Geary et al., 2000; Geary et al., 2008; Keeler & Swanson, 2001; Koontz & Berch, 1996; Mabbott & Bisanz, 2008; Montague & Applegate, 1993; Ostad, 1999; Passolunghi, 2011; Räsänen & Ahonen, 1995; Rousselle & Noël, 2007, 2008; Schuchardt et al., 2008; Swanson, 1993, 1994) compared the mathematical calculation abilities of students with MLD versus students with NLD-age or NLD-grade. Four (Fuchs & Fuchs, 2002; Lucangeli, Coi, & Bosco, 1997; Mabbott & Bisanz, 2008; Montague & Applegate, 1993) compared two groups of students on word problem solving, four (Mabbott & Bisanz, 2008; Ostad, 1997, 1999; Räsänen & Ahonen, 1995) on arithmetic strategies, and five (Desoete & Grégoire, 2006; Koontz & Berch, 1996; Mabbott & Bisanz, 2008; Mazzocco & Devlin, 2008; Rousselle & Noël, 2007) on number sense.

**Mathematical calculations.** Of the 17 studies that examined group differences between students with MLD and students with NLD-age or NLD-grade on mathematical calculation measures, 14 (Desoete & Roevers, 2002; Fletcher, 1985; Garrett et al., 2006; Geary et al., 2000; Geary et al., 2008; Keeler & Swanson, 2001; Koontz & Berch, 1996; Mabbott & Bisanz, 2008; Passolunghi, 2011; Rousselle & Noël, 2007, 2008; Schuchardt et al., 2008; Swanson, 1993, 1994) focused on the elementary level and the other 3 (Montague & Applegate, 1993; Ostad, 1999; Räsänen & Ahonen, 1995) targeted the secondary level.

At the elementary level, not surprising, students with MLD in second through fifth grades had significantly lower calculation scores than elementary students with NLD-age or NLD-grade (SMD \( M < -3.24, \text{ps} < .05 \) or .001) as measured by standardized mathematics tests including the Iowa Test of Basic Skills; the Test of Early Mathematics Abilities–Second Edition (Ginsburg & Baroody, 1990); the WIAT (Wechsler, 1992) Mathematics Reasoning; the Wechsler Individual Achievement Test–Second Edition–Abbreviated (WIAT-II-A; Wechsler, 2001) Numerical Operations; the Woodcock–Johnson Psycho-Educational Battery–Revised (WJ-R; Woodcock & Johnson, 1989) Calculations; the WRAT (J. Jastak & Jastak, 1978) Arithmetic; and the Wide Range Achievement Test–Third Edition (WRAT-III; Wilkinson, 1993) Arithmetic subtest.

At the secondary level, two studies (Ostad, 1999; Räsänen & Ahonen, 1995) included secondary students as a subset of participants and one study (Montague & Applegate, 1993) included solely secondary students; results revealed that the calculation mean scores of students with MLD continued to be significantly lower than students with NLD-age or NLD-grade on their standardized mathematics tests. Specifically, in the longitudinal study by Ostad, two groups of students with MLD (third and fifth grades in their initial years) scored lower on their standard
mathematics test and gave more incorrect answers in basic subtraction fact skills in both their initial years and two years later (SMD $M = -1.75$). Third to sixth graders with MLD scored significantly lower than the NLD-grade on the WRAT-R (S. Jastak & Wilkinson, 1984) and the RMAT (Finnish arithmetic test; Rääsänen, 1992; SMD $M = -1.37$, $ps < .001$; Rääsänen & Ahonen, 1995). In addition, middle school students with MLD (sixth to eighth grades) scored significantly lower than students with NLD-age or NLD-grade on the Wechsler Intelligence Scale for Children–Revised (WISC-R; Wechsler, 1974) Arithmetic subtest, the Woodcock–Johnson Psychoeducational Battery (Woodcock & Johnson, 1977) Calculation and Quantitative Concepts (SMD $M = -2.65$, $ps < .001$; Montague & Applegate, 1993). Thus, quantitative differences noted in the elementary grades persisted in the secondary level grades.

Word problem solving. Students with MLD demonstrated lower problem-solving performance than age- or grade-matched peers with NLD in four studies. In three of the four studies, elementary students in fourth or fifth grades with MLD demonstrated significantly lower mathematics problem solving (SMD $= -4.39$, $p < .001$), more computational and procedural errors (SMD $= 0.99$, $ps < .001$; Lucangeli et al., 1997), and lower scores on both complex story problems and real-world problems compared to students with NLD-grade (SMD $M = -0.53$; Fuchs & Fuchs, 2002). In one study involving sixth to eighth graders (Montague & Applegate, 1993), students with NLD-grade significantly outperformed students with MLD on six word problems on the Mathematical Problem Solving Assessment (Montague & Bos, 1990; SMD $= -0.63$, $p < .01$). Thus, significant group differences noticed in both the elementary and secondary grades on word problem-solving measures. Yet, in Mabbott and Bisanz’s (2008) study, the group mean differences was smaller (SMD $= -0.18$) than those of other studies and the two groups scored the same on tasks asking to prove why a certain product was a correct answer for a specified question.

Arithmetic strategies. In four studies, students with MLD used more counting rather than retrieval strategies compared to students with NLD-grade or NLD-grade. For example, students with MLD (average 11.40 years of age) used more counting string (“using a memorized string to produce the answer”; SMD $= 0.20$) and repeated addition strategies (“adding an operand the appropriate number of times”; SMD $= 0.83$) for multiplication computation problems, while exhibiting less retrieval (“solving a problem from memory”; SMD $= -0.54$) and less special trick strategies (“using previously learned problem-solving skills”; SMD $= -0.06$; Mabbott & Bisanz, 2008, p. 18). Furthermore, these elementary school students had a significantly lower number of addition (Ostad, 1997) and subtraction (Ostad, 1999) strategy variants (SMD $M = -3.57$, $ps < .001$) than students with NLD-grade. Specifically, students with MLD demonstrated a significantly lower number of changes in strategy use compared to students with NLD-grade (SMD $M = -2.91$, $ps < .001$; Ostad, 1997, 1999). Students with NLD-grade usually chose several different strategies, which increased in later grades, demonstrating substantial knowledge of strategies (Ostad, 1997). Students with MLD in Grades 3 through 6 also made significantly more errors than students with NLD-grade on all arithmetic error types of basic arithmetic (i.e., addition, subtraction, and multiplication); students with MLD made relatively greater algorithmic errors in addition (SMD = 0.99) and subtraction (SMD = 0.83; Rääsänen & Ahonen, 1995).

Number sense. In two (Desoete & Grégoire, 2006; Mazzocco & Devlin, 2008) of the five studies that compared number sense concepts, students with MLD scored significantly lower than students with NLD-age or NLD-grade. Specifically, third graders with MLD had significantly lower prenumerical (e.g., “number word sequence: counting forward to an upper bound up to 9, counting forward from a lower bound from 7”; p. 365) and numerical concepts (e.g., “estimation of size”) on the TEDI-MATH diagnostic assessment (Grégoire, Van Nieuwenhoven, & Noël, 2004; SMD $M = 1.02$, $ps < .05$; Desoete & Grégoire, 2006), and middle school students with MLD statistically significantly incorrectly ranked the quantity of items on all subtests of the Ranking Proportions Test (i.e., Decimals, Pictures, Fractions, and Mixed) and identified significantly fewer correct equivalences (e.g., “$\frac{20}{100} = 0.2$” and “$0.6 = \frac{60}{100}$”) than did students with NLD-grade (SMD $M = -1.99$, $ps < .05$ or .01; Mazzocco & Devlin, 2008). Despite the finding from these two studies, results from three other studies (Koontz & Berch, 1996; Mabbott & Bisanz, 2008; Rousselle & Noël, 2007) revealed that elementary school students with MLD and students with NLD-age or NLD-grade had nearly comparable number sense. The two groups of students with MLD and students with NLD-grade failed similarly in the geometrical congruity effect task (“difference between data in the congruent and incongruent conditions”; p. 380) and on physical and collection comparison tasks (“in the density condition, all sticks were of equal size and the density of elements was controlled in each pair” and “in the surface condition, the total surface area of the sticks was equated in each pair by reducing the size of the elements in the collection with more sticks”; p. 373; SMD $M = -0.28$; Rousselle & Noël, 2007). Students with MLD scored significantly lower only on the Arabic number task (SMD $M = -0.70$, $p < .01$).

The group mean differences were marginal ($0.5 < p < .10$) on identifying both letter and numerical stimuli (Koontz & Berch, 1996). The two groups showed small group
differences on the repeated addition comparison concept, commutativity (presentation of reverse multiplier and multiplicand), and related fact comparisons (presentation of the same multiplicand and increased or decreased multiplier by one; SMD $M = 0.15$; Mabbott & Bisanz, 2008).

In sum, the overall findings showed students with MLD's lower abilities with calculating, solving word problems, using increasingly mature arithmetic strategies, and understanding number sense compared to students with NLD-age or NLD-grade. For example, students with MLD used more repeated addition or counting string strategies rather than more mature, efficient retrieval strategies compared to students with NLD-age or NLD-grade. They also showed limited number word sequence, estimation, ranking ordering, and equivalence of rational numbers.

Of interest, the two groups' SMDs were smaller on the number sense variable compared to those on other mathematical topics including mathematical calculations, word problem solving, and arithmetic strategies. The two groups showed no significant differences comparing number concepts and identifying numbers. Also, on tasks asking to “prove” problems using why questions, both groups scored the same. Finally, the two groups performed comparably low on the geometric form such as congruity effect task and density and surface conditions.

**Students with MLD versus younger students with no LD matched on the mathematical ability of students with MLD (RQ3).** Of the 23 studies, 5 studies (Desoete & Roeyers, 2002; Keeler & Swanson, 2001; Mabbott & Bisanz, 2008; Rousselle & Noël, 2008; Swanson, 1993) compared the mathematical calculations of students with MLD to younger students with NLD-ability. One (Mabbott & Bisanz, 2008) compared the two groups of students on word problem solving, one (Mabbott & Bisanz, 2008) on arithmetic strategies, and two (Desoete & Grégoire, 2006; Mabbott & Bisanz, 2008) on number sense.

**Mathematical calculations.** Of the five studies comparing elementary school students with MLD to younger students with NLD-ability, three (Keeler & Swanson, 2001; Rousselle & Noël, 2008; Swanson, 1993) showed that the two groups were comparable on mathematical calculation measures. The two groups showed marginal differences on the WJ-R (Woodcock & Johnson, 1989) Calculation subtest, the WRAT (J. Jastak & Jastak, 1978) Arithmetic subtest, and simple addition verification tasks, using an approximate calculation process ("i.e., presenting two incorrect results for a given equation"); p. 500; Rousselle & Noël, 2008; SMD $M = 0.01$).

In the two other studies, significant group mean differences were exhibited. Specifically, third grade, or about 11-year-old, students with MLD had significantly lower calculation scores than second grade, or about 9-year-old, younger students with NLD-ability as measured by the KRT of mental computation (e.g., “$129 + 879 = ___$”). Also, students with MLD had significantly lower scores on the number system knowledge task (e.g., “add three tens to 61 and you have ___”; Cracco et al., 1995; SMD = −2.46, $p < .05$; Desoete & Roeyers, 2002) and the WRAT-III (Wilkinson, 1993) Arithmetic subtest (e.g., counting, performing written computations; SMD = −1.35, $p < .01$; Mabbott & Bisanz, 2008).

**Word problem solving.** Only one study (Mabbott & Bisanz, 2008) compared the word problem-solving abilities of students with MLD to younger students with NLD-ability. On the multiplication concept measure of word problems, students with MLD (average 11 years of age) exhibited slightly higher performances than younger students with NLD-ability (average 9 years of age; SMD = 0.40). The word problem tests included four questions: one with irrelevant information that should have been ignored; one with insufficient information for solving the problem; one requiring multiplication, addition, and comparison to solve a multistep problem; and one with combining a group number into a set (Mabbott & Bisanz, 2008). On another concept measure of proofs that required students to group manipulatives into appropriate sets to show justifications of the answers to multiplication problems, students with MLD also exhibited slightly higher performances than younger students with NLD-ability (SMD = 0.24).

**Arithmetic strategies.** Students with MLD (average 11 years of age) demonstrated slightly more limited use of arithmetic strategies than younger students with NLD-ability (average 9 years of age) while solving 28 simple multiplication problems (Mabbott & Bisanz, 2008). Students with MLD used slightly more repeated addition, counting string strategies, and procedural skills (SMD $M = 0.12$). However, they used slightly less retrieval and special tricks than younger students with NLD-ability (SMD $M = −0.09$).

**Number sense.** Third graders with MLD and second graders with NLD-ability mostly demonstrated no significant group differences on measures of prenumerical, numeral, and comparison number concepts (Desoete & Grégoire, 2006). The prenumerical skills of students with MLD and younger students with NLD-ability were not significantly different on any of the tasks of the TEDI-MATH diagnostic assessment (Grégoire et al., 2004; $ps > .05$). Regarding the numeral skills, the groups’ means were not significantly different on arithmetical operations and estimation of size of numerical tasks (SMD $M = 0.23$, $ps > .05$; Desoete & Grégoire, 2006). In addition, the two groups showed minimal differences on repeated addition comparison, commutativity comparison (e.g., “$8 \times 59 = 472$ paired with $59 \times 8 = 472$” or “$4 \times 64 = 256$ paired with $64 \times$
Long-term memory. Findings from Fletcher (1985) showed that there were no statistically significant group differences between students with MLD (average 10.04 years of age) and students with MLD/RLD (average 9.90 years of age) on both verbal and nonverbal tasks that required long-term retrieval (i.e., “number of items that have entered long-term storage and recalled on all subsequent trials”; p. 251; SMD $M = 0.35$, $ps > .05$).

In the Geary et al. (2000) study, the ability to gather information stored in long-term memory was measured by verbal tasks that consisted of articulating familiar words and nonwords. Geary et al. showed that second graders with MLD showed slightly faster familiar-word (“one-syllable words such as school-tree-cake and one-digit numbers such as 2, 9, 5”; p. 247) articulation speed than did second graders with MLD/RLD, once IQ and nonword (“one-syllable nonwords such as lote, dake, pog”; p. 247) articulation speed had been statistically controlled (SMD = −0.10 on word stimulus and −0.38 on number stimulus). However, when the nonword verbal task was provided, students with MLD showed slightly slower articulation speeds than students with MLD/RLD (SMD = 0.47).

Metacognition. Third graders with MLD did not significantly differ from third graders with MLD/RLD on the Evaluation and Prediction Assessment (EPA2000; De Clercq, Desoete, & Roeyers, 2000; SMD = 0.11, $p > .05$; Desoete & Roeyers, 2002). For example, the two groups demonstrated comparable metacognitive prediction (i.e., students predict whether or not they are likely to solve a particular problem) and evaluation skills (i.e., students judge how well they did without receiving feedback) with 80 mathematical problem-solving tasks (SMD $M = 0.13$, $ps > .05$).

In summary, the group differences were not significantly different across all cognitive measures of working memory, long-term memory, and metacognition abilities. Despite the marginal group differences on working memory, they showed the least marginal differences on visual-spatial tasks. Also, despite no significant group differences on both verbal and nonverbal tasks requiring long-term retrieval, students with MLD showed slower articulation speed than students with MLD/RLD after receiving the nonword stimulus. On metacognitive prediction and evaluation tasks with mathematical problem solving, the two groups showed comparable metacognitive capacity.

Students with MLD versus students with no LD matched on the same age or grade of students with MLD (RQ2). Of the 23 studies, 9 studies (Geary et al., 2000; Keeler & Swanson, 2001; Koontz & Berch, 1996; Mabbott & Bisanz, 2008; Passolunghi, 2011; Schuchardt et al., 2008; Siegel & Ryan, 1989; Swanson, 1993, 1994) compared the working memory of students with MLD to students with NLD matched on the age or grade of students with MLD. Six (Geary et al., 2008; Koontz & Berch, 1996; Mabbott & Bisanz, 2008; Mazzocco & Devlin, 2008; Passolunghi, 2011; Roussele & Noël, 2007) compared processing speed, three (Fletcher, 1985; Geary et al., 2000; Mabbott & Bisanz, 2008) examined long-term memory, and four (Desoete & Roeyers, 2002; Garrett et al., 2006; Lucangeli et al., 1997; Montague & Applegate, 1993) investigated...
the metacognitive abilities of students with MLD versus students with NLD-age or NLD-grade.

**Working memory.** Nine studies were located that focused on comparisons between MLD and NLD-age or NLD-grade. Elementary school students with MLD showed significantly limited visual-spatial working memory on mapping and directions, picture sequence, and visual matrix tasks (SMD = −0.68, ps < .05 or .01; Keeler & Swanson, 2001; Swanson, 1993, 1994).

Regarding the central executive working memory, studies reported contrasting findings depending on tasks. Students with MLD demonstrated significantly lower central executive working memory on Digit Sentence Span, Backward Digit Span, the operation span task, the listening span for target words, and series than students with NLD-age (average 9.63 years of age; SMD = −1.00, ps < .05 or .01; Mabbott & Bisanz, 2008; Passolunghi, 2011). However, students with MLD did not have significantly lower scores than students with NLD-age on sentence tasks (SMD = −0.42; Siegel & Ryan, 1989).

Regarding phonological loop capacity, studies reported contrasting findings. In Koontz and Berch (1996), students with MLD demonstrated limits in their phonological loop capacity with significantly smaller digit spans and letter spans than students with NLD-grade (SMD = −1.04, ps < .05 or .01). However, the two groups did not differ significantly on the word forward span, the WISC-R (Wechsler, 1974) Digit Span, or the syllable word and non-word (SMD = −0.42, ps > .05; Passolunghi, 2011; Schuchardt et al., 2008) tasks.

**Processing speed.** All six studies that examined differences in processing speed between students with MLD and students with NLD-age or NLD-grade reported limited processing speed of students with MLD. For example, elementary school students with MLD-grade showed significantly longer reaction times on the rapid automatized naming task, the Arabic number comparison task (i.e., symbolic number magnitude), and both digits and letters stimulus tasks (SMD = 0.86, ps < .10 or .02; Geary et al., 2008; Koontz & Berch, 1996; Rousselle & Noël, 2007). They also demonstrated limited processing speed on other tasks including correct retrieval trial (SMD = 0.83) and backup procedures (SMD = 3.58; Mabbott & Bisanz, 2008). In addition, students with MLD in middle school levels showed significantly longer response times to complete the Picture subtest than their peers with NLD-grade (SMD = 1.10, p < .01; Mazzocco & Devlin, 2008). On the tasks of physical size comparison, collection comparison (density and surface condition) in the ratio set, and congruity effect between physical and numerical dimensions, the reaction times of students with MLD were slightly slower than those of students with NLD-grade (SMD = 0.50; Rousselle & Noël, 2007).

**Long-term memory.** Studies examined elementary school students’ responses for both verbal and nonverbal tasks. In the two studies that measured long-term memory involving verbal tasks, there were no significant group differences on all verbal tasks (SMD = −0.19, ps > .05; Fletcher, 1985). The two groups also demonstrated comparable articulation speeds on familiar word stimuli tasks (SMD = 0.79, p > .05; Geary et al., 2000).

On nonverbal tasks, however, students with MLD demonstrated significantly lower scores than students with NLD-age (SMD = −1.37, p < .05; Fletcher, 1985). Students with MLD also had slower articulation speeds than students with NLD-grade on the nonword stimulus task (SMD = 0.80; Geary et al., 2000). Moreover, students with MLD exhibited lower retrieval and backup accuracy than students with NLD-age (SMD = −0.23; Mabbott & Bisanz, 2008). Students with MLD scored significantly lower than students with NLD-age or NLD-grade on measures of metacognition and used a limited number of metacognitive strategies in the four studies. For example, students with MLD in Grade 3 predicted and evaluated significantly less well than their peers with NLD-grade on all prediction and evaluation tasks of mathematical problem solving (SMD = −1.87, ps < .05; Desoete & Roeyers, 2002).

Regarding the task difficulties, students with MLD had significantly lower scores on both prediction and evaluation tasks designed for Grades 1, 2, and 3 than did students with NLD-grade (SMD = −0.30, ps < .05; Desoete & Roeyers, 2002). On tasks designed for Grade 4, however, students with MLD had significantly better scores on prediction (SMD = 0.41, p < .05) and slightly better scores on evaluation (SMD = 0.01, p > .05) than students with NLD-grade (Desoete & Roeyers, 2002).

Fifth graders with MLD with poor problem-solving skills had significantly lower levels of metacognitive awareness than fifth graders with NLD-grade (SMD = −0.73, p < .001; Lucangeli et al., 1997). They were more likely to believe that the size of numbers in a problem was an indicator of the difficulty of the problem (SMD = 1.10, p < .001; Lucangeli et al., 1997). Students with MLD in Grades 6 through 8 reported significantly fewer problem representation strategies than their peers with NLD-grade (SMD = −1.40, p < .01; Montague & Applegate, 1993); yet the two groups demonstrated no significant differences for the number of strategies regarding strategy knowledge, strategy use, and strategy control as well as for the quality of strategies with IQ as the covariate on the mathematical problem solving assessment (SMD = −0.51; Montague & Applegate, 1993).

In brief, the overall findings showed that students with MLD demonstrated significant difficulties with working memory, long-term memory, processing speed, and
metacognition compared to students with NLD-age or NLD-grade. In particular, students with MLD performed significantly less well than NLD-age or NLD-grade on visual-spatial working memory tasks. However, the findings were mixed regarding central executive and phonological loop working memory. Students with MLD showed significantly limited central executive and phonological loop capacities on most tasks but not on sentence spans and word forward spans. In addition, of the long-term memory tasks, student with MLD and students with NLD-age or NLD-grade did not show significant group differences on verbal-related tasks, yet they showed significantly limited long-term memory on nonverbal tasks.

**Students with MLD versus younger students with no LD matched on the mathematical ability of students with MLD (RQ3).** Of the 23 studies, 3 (Keeler & Swanson, 2001; Mabbott & Bisanz, 2008; Swanson, 1993) compared students with MLD to younger students with NLD-ability on working memory. One (Mabbott & Bisanz, 2008) examined differences in processing speed, one (Mabbott & Bisanz, 2008) differences in long-term memory, and one (Desoete & Roeyers, 2002) differences in metacognition between the two groups of students.

**Working memory.** In the three studies that examined differences in working memory, the two groups of elementary school students showed no significant group differences on central executive working memory. For example, students with MLD and younger students with NLD-ability performed similarly on the Digit Sentence Span task (SMD = 0.16, p > .01; Keeler & Swanson, 2001). In addition, they showed comparable spans on operation span and the WISC-III (Wechsler, 1991) Backward Digit Span subtest (Mabbott & Bisanz, 2008).

Regarding verbal and visual-spatial tasks, findings differed depending on the types of tasks. For example, students with MLD had significantly higher scores on the number knowledge, mental arithmetic, and procedural calculation (SMD = −0.44, ps < .05) and significantly lower scores on number system knowledge and procedural calculation (SMD = −0.40, ps < .05) than younger students with NLD-ability; no significant group differences were found on other prediction (i.e., numeral and operation symbol comprehension and word problem) and evaluation tasks (i.e., numeral and operation symbol comprehension, mental arithmetic, and word problem; ps > .05).

In short, the overall findings were mixed in terms of statistically significant differences between the two groups. There were no significant group differences on central executive working memory. Regarding verbal and visual-spatial tasks, findings differed depending on the types of tasks. Students with MLD demonstrated slightly faster processing speed for backup procedures. For long-term memory, students with MLD exhibited shorter latencies for backup procedures and slightly better long-term memory on retrieval procedures than younger NLD. However, findings on metacognition differed depending on the types of tasks. Students with MLD demonstrated significantly limited abilities with metacognitive tasks of prediction and evaluation for number system knowledge and procedural calculation tasks.

**Discussion**

The purpose of this article was to synthesize published articles that compared the mathematical and cognitive performances of students with MLD to students with MLD/RLD, students with NLD-age or NLD-grade, and younger students.

Long-term memory. Only one study (Mabbott & Bisanz, 2008) compared the long-term memory capacity of elementary school students with MLD and their younger peers with NLD-ability. MLD students showed slightly better long-term memory on retrieval and backup procedures than ability-matched younger students with NLD (SMD M = 0.49).

Metacognition. No group differences were found between third graders with MLD and second graders with NLD-ability on any prediction and evaluation skills on the EPA2000 (De Clercq et al., 2000; SMD M = −0.20; Desoete & Roeyers, 2002). Moreover, there were no significant group differences on prediction and evaluation tasks for the levels of Grades 2, 3, and 4 (SMD M = −0.11, ps > .05). However, students with MLD had significantly lower prediction and evaluation scores for the easiest task for Grade 1 level Grade 1 than younger students with NLD-ability (SMD M = −0.71, ps < .05).

Regarding different mathematical problem-solving tasks, students with MLD showed significantly lower prediction on the number knowledge, mental arithmetic, and procedural calculation (SMD M = −0.44, ps < .05) and significantly lower evaluation on number system knowledge and procedural calculation (SMD M = −0.40, ps < .05) than younger students with NLD-ability; no significant group differences were found on other prediction (i.e., numeral and operation symbol comprehension and word problem) and evaluation tasks (i.e., numeral and operation symbol comprehension, mental arithmetic, and word problem; ps > .05).

In short, the overall findings were mixed in terms of statistically significant differences between the two groups. There were no significant group differences on central executive working memory. Regarding verbal and visual-spatial tasks, findings differed depending on the types of tasks. Students with MLD demonstrated slightly faster processing speed for backup procedures. For long-term memory, students with MLD exhibited shorter latencies for backup procedures and slightly better long-term memory on retrieval procedures than younger NLD. However, findings on metacognition differed depending on the types of tasks. Students with MLD demonstrated significantly limited abilities with metacognitive tasks of prediction and evaluation for number system knowledge and procedural calculation tasks.
with NLD-ability. By comparing students with MLD to students with MLD/RLD, we examined the similarities and differences in mathematical and cognitive performances of the two groups. By comparing students with MLD with students with NLD-age or NLD-grade and with younger students with NLD-ability, we determined the relative performance within the mathematical and cognitive topics.

Taken together, these findings could provide a better theoretical understanding of MLD and hypotheses of serious mathematical deficits. In addition, findings could support implications for the development of mathematical interventions for students with MLD. Both the mathematical and cognitive performances of students with MLD warrant examination.

**Mathematical Performance**

Research Question 1 addressed comparisons between students with MLD and students with MLD/RLD. The findings for the two groups did not show significant group differences on mathematical calculation measures. However, students with MLD exhibited higher word problem solving and better arithmetic fact strategy than students with MLD/RLD. These findings are similar to those discussed by Geary (2004), who in his article on “Mathematics and Learning Disabilities” revealed that across studies (e.g., Ostad, 1997, 1999), students with MLD/RLD and MLD only continued to struggle with mathematical calculations, especially computing facts.

Especially regarding word problem solving, the two groups showed relatively larger differences on the text-based tasks than on the arithmetic component-based tasks. Students with MLD exhibited higher word problem-solving scores than students with MLD/RLD with mainly moderate and large SMDs except for the operation dimension on complex story problems (SMD = 0.19). These results indicate that students with MLD who do not have difficulties in reading have better abilities in solving mathematical word problems compared to students with MLD/RLD, who have disabilities in reading as well as mathematics. Jordan (2007) pointed out that word problem solving that is dependent on numerical verbal skills differentiates students with MLD from students with MLD/RLD. Bryant, Smith, and Bryant (2008) also stated that students’ struggles with word problem solving mainly come from failures in reading the problem, understanding the meaning of the sentences, and understanding what is asked. The two groups’ considerable differences on the problem-solving dimension (SMD $M = 0.96$) as compared to the operation dimension (SMD $M = 0.35$), as measured by complex story and real-world problem tests, also indicate distinctively stronger word problem-solving capacity of students with MLD and less distinctive differences in the mathematical operation. Thus, teachers and intervention developers need to pay particular attention to teaching mathematical calculations, particularly basic facts and providing more intensive instruction on word problem solving.

Research Question 2 addressed comparisons between students with MLD and students with NLD-age or NLD-grade. In most studies, students with NLD-age or NLD-grade outperformed students with MLD across mathematical calculations, word problem solving, arithmetic strategies, and number sense in the elementary and secondary level grades. Especially, there were significant group differences on all the measures of mathematical calculations and arithmetic strategies. The significantly lower mathematical calculations of students with MLD support the definition of MLD in IDEIA (2004), where students with a specific LD have limitations in their mathematical calculations. These findings are also consistent with the identified mathematical behaviors examined by Bryant et al. (2008) and Bryant, Bryant, and Hammill (2000), where students with MLD in elementary and secondary school years exhibited difficulty with solving multidigit calculations, word problems, and multistep problems, the use of effective counting strategies to calculate answers to arithmetic problems, and understanding the commutative property (Bryant et al., 2000; Bryant et al., 2008). In addition, the finding showed that students with MLD used more repeated addition and counting string strategies, which are viewed as “immature strategies” (Geary, 2004), rather than applying automatic retrieval strategies for their mathematical calculations, which tends to happen in second grade. Students with MLDs’ lack of developmentally mature counting strategies such as retrieval strategies could be one source of their lower mathematical computation achievements than their peers with NLD-age or NLD-grade and a defining quality of MLD (Geary, 2010, 2011).

Noticeably, the finding on word problem-solving and number sense measures differed depending on tasks. Students with MLD showed significantly lower word-problem solving, number word sequence, estimation, and rational numbers than students with NLD-age or NLD-grade. However, the two groups showed comparable mathematical performances on problem-solving tasks, asking to “prove” problems, and on the geometric form of congruity effect and physical and collection comparison tasks. These findings indicate that both groups similarly struggled with the mathematical topics of mathematical reasoning and geometry concepts and skills during their primary school years.

With the importance of grade-level mathematics and standards (Council of Chief State School Officers & National Governors’ Association, 2010), mathematical reasoning is an essential component for all grade levels. In addition, as one of the foundational skills for success in algebra (National Mathematics Advisory Panel, 2008), difficulty with geometry reminds us of the importance of teaching geometry concepts and skills to mitigate the effects of MLD in later grades.
Research Question 3 addressed comparisons between students with MLD and younger students with NLD-ability. Overall, findings revealed mixed outcomes on mathematical performance. There were no significant group differences on most measures in mathematical calculations (Keeler & Swanson, 2001; Rousselle & Noël, 2008; Swanson, 1993), arithmetic strategies (Mabbott & Bisanz, 2008), and prenumerical concepts (Desoete & Grégoire, 2006; Mabbott & Bisanz, 2008), indicating developmental delays. However, there were contrasting findings on mathematical calculation measures. Although the two groups were comparable on most standardized mathematics tests, students with MLD showed significantly lower scores on mental computation tasks than younger students with NLD-ability. This limited mental computation skills could have occurred because students with MLD significantly lacked the mature computation skills of memory-based retrieval strategies than students with NLD-ability (Mabbott & Bisanz, 2008). Especially, as the problems become more difficult and cognitively demanding, students with MLD were outperformed by younger students with NLD-ability because students with MLD are lacking the strong foundation, which is necessary for more complex mathematics.

In addition, among the number sense tasks, the significantly better knowledge of numerical systems of students with MLD than students with NLD-ability was noticed (Desoete & Grégoire, 2006). This finding indicated that younger students with NLD-ability might not have learned the comparable mathematical content knowledge to the same degree as older students with MLD; this is supported by the finding that the two groups did not have significant differences on prenumerical concepts (e.g., “counting forward or backward, counting by 2 or 10, and classification of numbers”; p. 365) and had distinctive differences only on the knowledge of numerical systems (e.g., “Arab numerals, number words, and base-ten system”; p. 365).

**Cognitive Performance**

This study also focused on the cognitive performance of students with MLD. The working memory, processing speed, long-term memory, and metacognition of students with MLD were compared to those of students with MLD/RLD, students with NLD-age or NLD-grade, and younger students with NLD-ability on each measure.

Research Question 1 addressed comparisons between students with MLD and students with MLD/RLD. In general, there were no significant group differences across all cognitive topics. Noticeably, the relatively large SMDs on phonological loop and central executive tasks compared to those on visual-spatial tasks reflect students with MLD’s relatively stronger phonological loop (i.e., storage for verbal information) and central executive working memory (i.e., coordinating phonological loop and visual-spatial component) than visual-spatial working memory (i.e., storage for visual-spatial images; Baddeley & Hitch, 1974; Baddeley & Logie, 1999). The finding showed that the two groups had the least marginal differences on the visual-spatial tasks. The finding of Fuchs and Fuchs’s (2002) study reinforces the role of phonological loop and central executive functioning in solving word problems. The better central executive capacity and phonological loop working memory of students with MLD than students with MLD/RLD, who scored lower on problem-solving tasks, suggests that solving word problems requires the functions of phonological loop and central executive. Zheng et al. (2011) also found that central executive processing was the strongest predictor of word problem solving. Because central executive functioning requires inhibiting irrelevant information when integrating incoming and previously encoded information in the working memory (Bull & Espy, 2006), better executive functioning theoretically can facilitate the problem-solving process.

Research Question 2 addressed comparisons between students with MLD and students with NLD-age or NLD-grade. The findings showed significant group differences across all the cognitive topics in most studies. Regarding working memory tasks, students with MLD showed significantly lower visual-spatial working memory than students with NLD-age or NLD-grade. The profound lack of visual-spatial capacities of students with MLD has been highlighted in previous studies (Geary, 1993, 2004, 2010, 2011; Swanson & Jerman, 2006). Students with MLD also had significantly limited central executive and phonological loop working memory on number-related tasks (Mabbott & Bisanz, 2008; Passolunghi, 2011). Of interest, within the findings on central executive working memory, Siegel and Ryan (1989) reported that students with MLD exhibited no significant differences on the non-numerical verbal working memory tasks (e.g., sentences tasks), yet significant deficits on the numerical verbal working memory tasks (e.g., counting tasks). This finding is consistent with other studies that support the deficit of domain-specific knowledge that is related to numbers among students with MLD (Raghubar, Barnes, & Hecht, 2010). MLD students were likely not to have a deficit on a language-related working memory task (e.g., phonological loop tasks) that measures processes similar to reading, yet they had limitations on a working memory task that includes counting and remembering the products of those counts (Siegel & Ryan, 1989).

The findings on long-term memory tasks between students with MLD and students with NLD-age or NLD-grade also imply domain-specific deficits of students with MLD. Students with MLD demonstrated significantly limited storage and retrieval of long-term memory compared to students with NLD-age or NLD-grade, but only on nonverbal tasks, not on verbal tasks. These nonverbal-related long-term memory limitations are consistent with the domain-specific
working memory deficits of students with MLD, who had significant limitations on visual-spatial working memory.

Research Question 3 addressed comparisons between students with MLD and younger students with NLD-ability. The two groups showed no significant group differences on all central executive working memory tasks and some verbal (e.g., phrase sequence) and visual-spatial tasks (e.g., mapping/directions). In addition, the two groups showed no significant group differences on such tasks as numeral and operation symbol comprehension, yet significant differences on number system knowledge and procedural calculation. This finding indicated that working memory and metacognition could be subject to developmental delays or maturational lag (Desoete & Roeyers, 2002), supported by there being no significant differences between students with MLD and younger students with NLD-ability on many working memory (i.e., central executive) and metacognition measures.

**Limitations**

There are three limitations that need to be considered when interpreting the findings of this study. First, it was difficult to fully compare the aspects of mathematical and cognitive performances across all the mathematical and cognitive topics for the three comparison groups because of the limited focus of interest across the studies. All studies compared students with MLD versus students with NLD-age or NLD-grade, and few studies compared students with MLD versus students with MLD/RLD (n = 7) and students with MLD versus younger students with NLD-ability (n = 6) on either mathematical or cognitive topics. Thus, there is still limited understanding on how students with MLD differ from students with MLD/RLD and from younger students with NLD-ability. In addition, studies did not equally compare specific mathematical or cognitive topics. That is, many studies focused mainly on mathematical calculations of the four mathematical topics. No article compared number sense between students with MLD and students with MLD/RLD. In addition, studies mainly focused on working memory in the four cognitive topics. No article compared processing speed between students with MLD and students with MLD/RLD.

Second, regarding the definitions and criteria of students with MLD, studies used various operational definitions for the identification of students with MLD with different identification model and cutoff points. Regarding models and criteria for identifying students with MLD, 20 of the 23 studies applied the low-achievement model. Of the 20 studies, 8 reported using LA on standardized mathematics tests (e.g., at or below the 15th through 31st percentile) and low or average achievement on reading tests (e.g., at or above the 15th through 40th percentile) along with low or average IQ scores (e.g., at least 80 or 85); 7 used LA on standardized mathematics tests and low or average achievement on IQ tests; and the remaining 5 used LA only on standardized mathematics tests. In addition, 3 of the 23 studies applied the IQ–achievement discrepancy model (e.g., a critical discrepancy of 1.2 SD between IQ and an overall achievement score).

Based on some commonly limited working memory and procedural development in arithmetic facts, studies have been using various criteria to define students with MLD (Geary et al., in press). The inconsistent use of cutoff scores in determining students with MLD is a critical issue because it could broadly include groups of students with MLD who are in various mathematical and cognitive skills (Murphy et al., 2007). As a result, the inconsistent cutoff criteria might have included the range of low-achieving students in the various distributions of test scores (Büttner & Hasselhorn, 2011; Geary et al., in press). Also, there were differences across studies because of a large range of years, 1975 and 2011. In addition, considering that 13 of 23 studies were conducted outside the United States, this synthesis included studies in the range of different operational definitions and criteria for identifying MLD students according to each country’s laws.

Third, it was difficult to generalize the findings across all grades. Out of the 23 studies, most (n = 18) focused on elementary school students, and a limited number of studies (n = 5) focused on secondary school (i.e., middle school) students. Because no studies targeted the high school student population, the findings should be interpreted carefully with regard to secondary school students.

**Future Research**

More studies need to focus on various mathematical and cognitive topics across comparison groups of students with MLD, students with MLD/RLD, students with NLD-age or NLD-grade, and younger students with NLD-ability. Most studies compared students with MLD and students with NLD-age or NLD-grade, yet there is still a lack of understanding of how students with MLD differ from students with MLD/RLD and younger students with NLD-ability. Across these comparison groups, more studies need to focus on word problem solving, arithmetic strategies, and number sense as well as processing speed, long-term memory, and metacognition.

In addition, more studies should focus on secondary school students to see age variants across different topics. As seen in this synthesis, most studies included elementary school students, limiting the generalization for secondary, in particular high school, students. Secondary school students with MLD, students with MLD/RLD, students with NLD-age or NLD-grade, and younger students with NLD-ability may demonstrate different aspects of mathematical performance in their grade-specific mathematics topics and show different features of cognitive performance according to their cognitive development.
Last, this synthesis of the findings could inform the field about the performances of students with MLD on mathematical and cognitive topics for future research in developing mathematical interventions for them. Within the specific mathematical topics, the relatively higher or lower mathematical calculations, word problem solving, arithmetic strategies, and number sense of students with MLD compared to the other groups can tell the specific mathematical areas that need more intensive interventions than other areas. In addition, the relatively higher or lower working memory, processing speed, long-term memory, and metacognition of students with MLD than those of students with MLD/RLD, students with NLD-age or NLD-grade, and younger students with NLD-ability could demonstrate the specific sources of mediators where students with MLD struggle with beyond the mathematical topics.

**Practical Implications**

There are three practical implications of the findings from this synthesis for educational purposes. First, put simply, defining students with MLD is challenging. Mazzocco and Myers (2003) pointed out the complexities of defining students with MLD and found that different groups of students were included within the category of MLD as assessed by different identification measures. In this present synthesis, studies also applied different operational definitions and criteria for determining participants with MLD: Studies used a low-achievement, criteria-based model with different cutoff points, ranging from below the 35th percentile to below the 10th percentile on standardized mathematics tests, or used an IQ–achievement discrepancy model for students who had been identified by the school district as having a LD and had a mathematics goal on their IEP or attended a special school for children with LD; yet not all studies included district- or teacher-identified students with LD. The different identification measures and methods could include a range of different groups of students with MLD and demonstrate various degrees of severity and aspects of mathematical and cognitive performances within the same groups of students with MLD. Therefore, researchers must reach consensus on the definition of MLD and MLD/RLD and ways to operationalize the definitions by taking into account findings from an array of studies, which were described in this synthesis.

Second, the use of different terms for referring to students with MLD reflects how researchers and educators in the field of special education have been broadly including students with MLD. Different terms were used in each study for MLD. For example, MLD was used interchangeably with mathematics disabilities, arithmetic disabled or disabilities, low problem solving, mathematically disabled, and specific disorders of arithmetical skills.

Finally, comparing students with MLD to different groups of students (viz., students with MLD/RLD, students with NLD-age or NLD-grade, and younger students with NLD-ability) on both mathematical and cognitive topics, educators can understand the source of the etiology of MLD and how students with MLD respond similarly to or differently from other students. The findings of this study provide detailed information about the mathematical and cognitive topics in which students with MLD have more struggles compared to other groups. The deficits in certain mathematical (e.g., fact strategies, problem solving) and cognitive topics (e.g., working memory, processing speed) could provide indicators of MLD and support the notion of how we define students with MLD.

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**References**

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